647510 **BSc3MatC0301** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination
Oct/Nov. -2019 (Old Course)
MATHEMATICS(CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

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1 (A) Answer any one: **(07)**

(1) If $\{a_n\},\{b_n\}$ are two sequences such that
$\lim_{n\to\infty} a_n = a, \lim_{n\to\infty} b_n = b$ then prove that $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = \frac{a}{b}$.

(2) Prove that: $\lim_{n\to\infty} \sqrt[n]{a} = 1; a > 0.$

(B) Answer any one: **(04)**

(1) Prove that the sequence $\{S_n\}$ defined by
$S_{n+1} = \frac{1}{2}\left(S_n + \frac{9}{S_n}\right), n \geq 1, S_1 > 0$ converges to 3.

(2) Show that $\lim_{n\to\infty}\frac{1}{n}\left(1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \cdots n^{\frac{1}{n}}\right) = 1.$

(C) Answer any one: **(03)**

(1) Define: Convergence of sequence and discuss the convergence of sequence $\left\{\frac{5n+(-1)^n}{2n+5}\right\}$.

(2) Define: Cauchy sequence and state the Cauchy's general Principle of convergence.

2 (A) Answer any one: **(07)**

(1) Using Raabe's test, discuss the convergence of $\sum\frac{4\cdot7\cdot10\cdots\cdots(3n+1)}{1\cdot2\cdot3\cdots\cdots n}x^n$

(2) State: D'Alembert's ratio test and discuss the convergence of

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \cdots$$

(B) Answer any one: **(04)**

(1) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty}\frac{(-3)^n x^n}{\sqrt{n+1}}$

(2) Define: p-series and test the convergence of series $\sum\sqrt{\frac{1+2^n}{1+3^n}}$.

(C) Answer any one: **(03)**

(1) Define: Alternating series and show that the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$ is convergent.

(2) Discuss the convergence of the series $1 + 2^2x + 3^2x^2 + 4^2x^3 + \cdots$

3 (A) Answer any one: **(07)**

(1) If $\acute{f}$ and $\acute{g}$ are vector functions defined on the domain D whose first order derivatives exist and $\emptyset$ is a scalar function on D whose first order partial derivative also exist then prove that:
curl $(\emptyset\acute{f}) = \emptyset curl\acute{f} + (grad\emptyset) \times \acute{f}$

(2) If $\acute{f}$ and $\acute{g}$ are irrotational functions on D then show that $\acute{f} \times \acute{g}$ is a solenoidal function.

(B) Answer any one: **(04)**

(1) Prove: (i) $(\acute{r} \times \acute{a}) = 0$(ii) $curl(\acute{r} \times \acute{a}) = -2\acute{a}$, where $\acute{a}$ is a constant vector.

(2) In usual notations prove that: $(r^n\acute{r}) = (n+3)r^n$.

(C) Answer any one: **(03)**

(1) Find the unit normal at the surface $x^2y + 2xz = 4$ at the point $(2,-2,3)$.

(2) If $\acute{f} = (x^3, y^3, z^3)$ then find $curl\acute{f}$.

4 (A) Answer any one: **(07)**

(1) Define: Double Integral. Prove that :

$\iint\limits_R (1-x-y)^3 x^{\frac{1}{2}} y^{\frac{1}{2}} dxdy = \frac{\pi}{480}$; where R is triangle whose vertices are $(0,0), (0,1), (1,0)$.

(2) Evaluate: $\iiint\limits_R \frac{dxdydz}{(1+x+y+z)^3}$, where R is bounded by $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

(B) Answer any one: **(04)**

(1) By changing variables in to polar coordinates find the value of $\iint\limits_R (1-x^2-y^2)dxdy$, where R is $x^2 + y^2 \leq 1$.

(2) Evaluate: $\int_0^1 \int_0^{x^2} \int_0^{x+y} (2x-y-z)\, dzdydx$

(C) Answer any one: **(03)**

(1) Change the order of integration in $\int_0^a \int_y^a f(x,y)dxdy$.

(2) Change in cylindrical co-ordinates

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{1+x+y} (x^2-y^2)\, dzdydx$$

5 (A) Answer any one: **(07)**

(1) State and prove Green's theorem for a plane.

(2) In usual notation prove that:

$$\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}} dx\, , p > 0, q > 0.$$

(B) Answer any one: **(04)**

(1) If $V = y\acute{\imath} + z\acute{\jmath}$ and S is a part of $2x + 2y + z = 2$ then find $\iint\limits_S V_n\, d\sigma$.

(2) Find: $\iint\limits_S x^2 dydz + y^2 dzdx + 2z(xy-x-y)dxdy$, where $S: 0 \leq x, y, z \leq 1$a solid surface.

(C) Answer any one: **(03)**

(1) Define: Line Integral. Find $\int_{(1,1)}^{(2,3)} xdy$.

(2) In usual notation, prove that: $p\beta(p, q+1) = q\beta(p+1, q)$.

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